

Definition of Signal

😊 A signal is a function of time representing a physical variable, e.g. voltage, current, spring displacement, share market prices, number of student asleep in the Lab, cash in the bank account.

😊 Typically we will use a mathematical function such as $f(t)$, $u(t)$ or $y(t)$ to describe a signal which is a continuous function of time.

Signal Classification

Signals may be classified into:

1. Continuous-time and discrete-time signals
2. Analogue and digital signals
3. Periodic and aperiodic signals
4. Energy and power signals
5. Deterministic and probabilistic signals
6. Causal and non-causal
7. Even and Odd signals

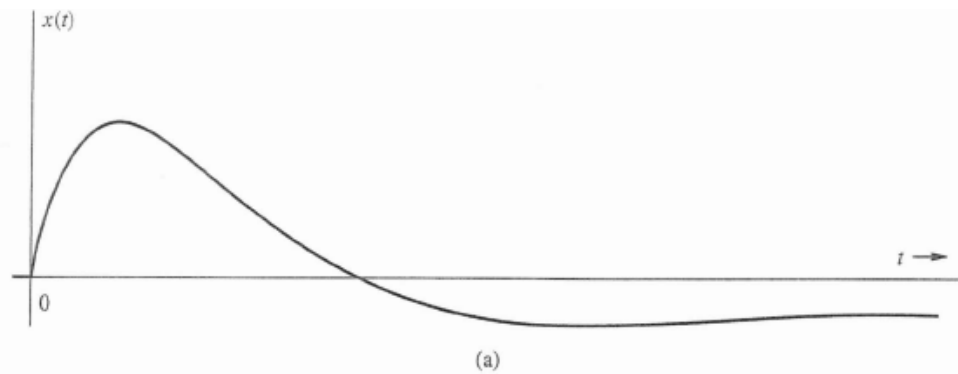
Classification of signals

😊 A **continuous-time** (or analog) signal exists at all instants of time. The real world consists of continuous signals, and are usually written as a function of t .

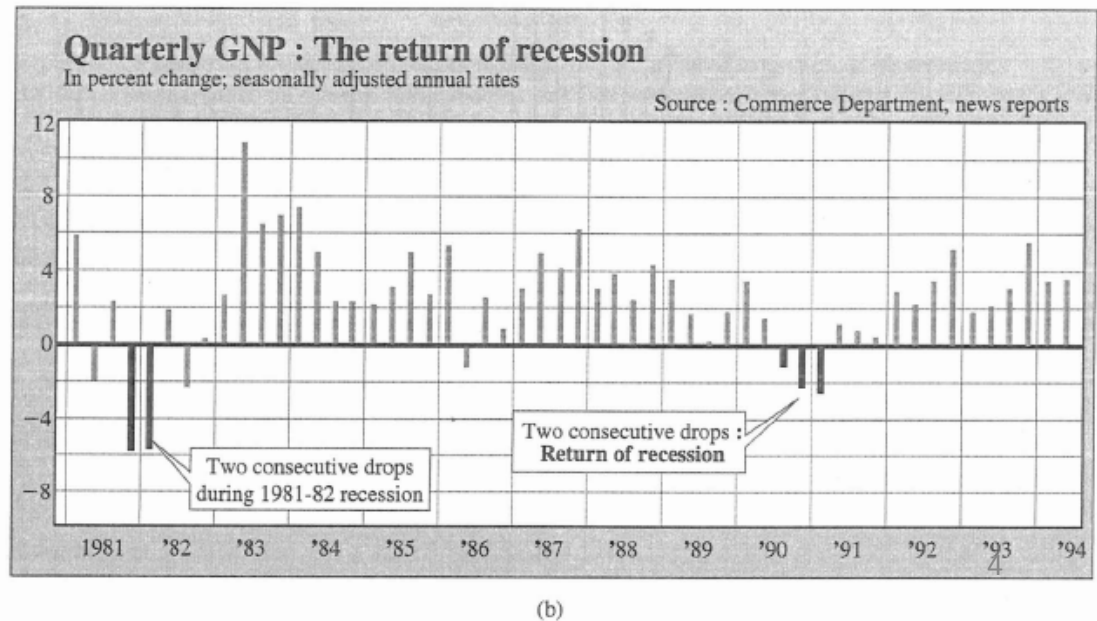
😊 A **discrete-time** signal exists only at discrete instants of time and is usually derived from a continuous signal by the process of sampling, e.g. measuring the temperature at 3 o'clock in the afternoon.

Signal Classification- Continuous vs Discrete

Continuous-time

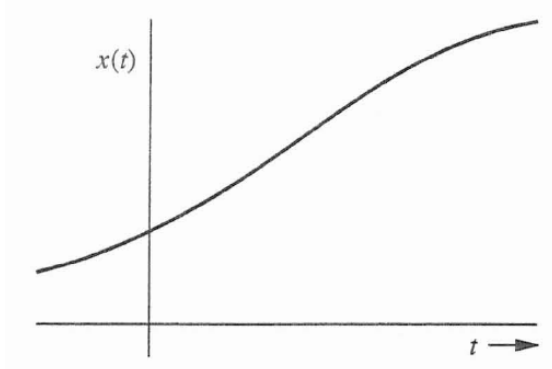


Discrete-time

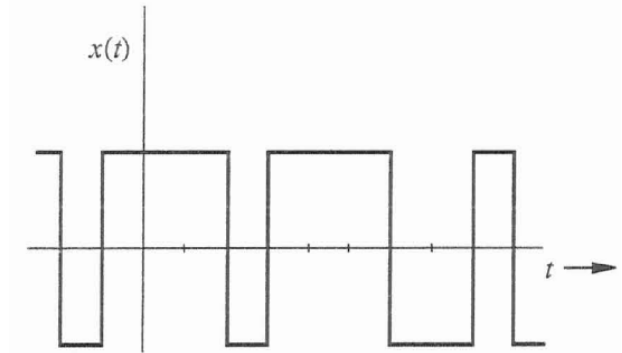


Signal Classification- Analogue vs Digital

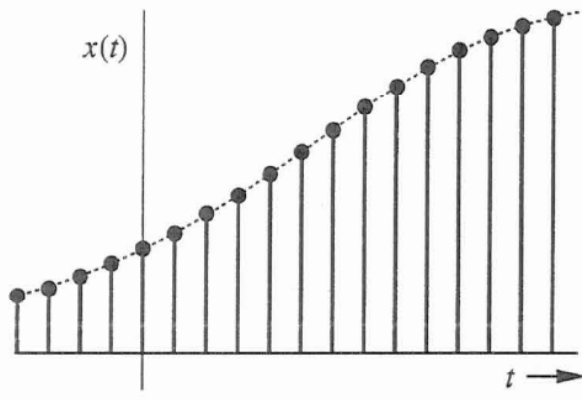
Analogue, continuous



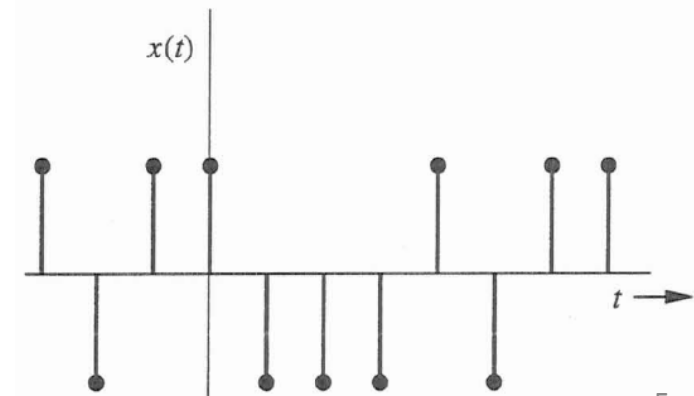
Digital, continuous



Analogue, discrete



Digital, discrete



Signal Classification

- Periodic and non-periodic signals
- A signal is periodic if

$$x(t) = x(t + T)$$

where T is the period.

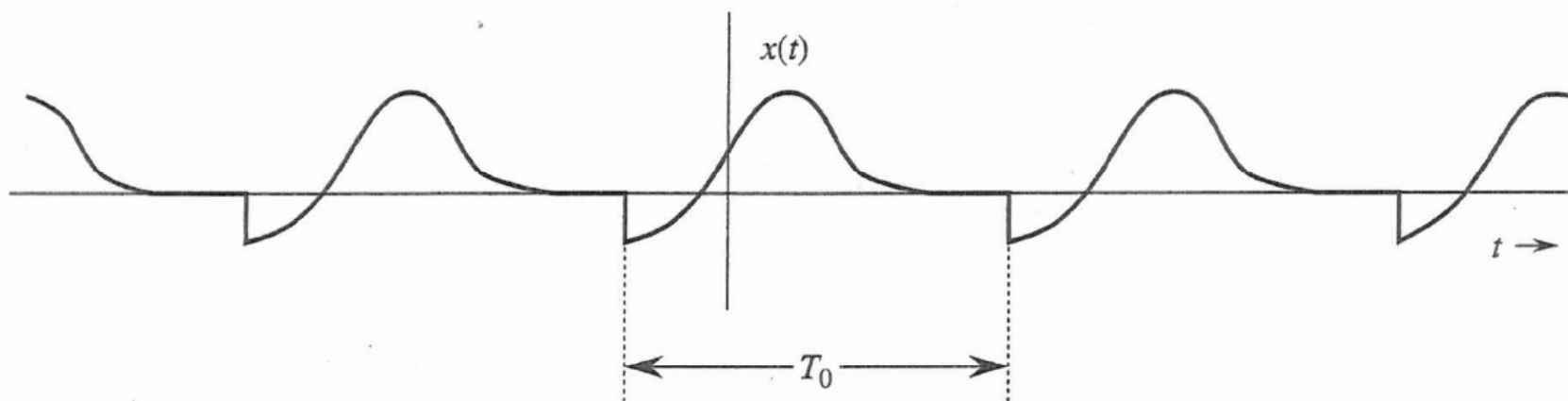
□ Non-periodic signals are those which do not follow this equation

$$x(t) \neq x(t + T)$$

A signal $x(t)$ is said to be periodic if for some positive constant T_0

$$x(t) = x(t+T_0) \text{ for all } t$$

The smallest value of T_0 that satisfies the periodicity condition of this equation is the *fundamental period* of $x(t)$.



Signal Classification- Energy v/s Power

- Energy of a signal $x(t)$ is given by:

$$E_x = \int_{-\infty}^{\infty} |x(t)|^2 dt$$

- Power of a signal $x(t)$ is given by:

$$P_x = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 dt$$

- A signal is Energy signal if $0 < E_x < \infty$
- A signal is Power signal if $0 < P_x < \infty$

Energy Signal and Power Signal

- Consider applying a time varying voltage $v(t)$ to a resistor,

the instantaneous average power drops across the resistor is,

$$p(t) = v^2(t)/R$$

or

$$p(t) = i^2(t) \times R$$

Power Signal

- For the purposes of signal classification the average power of a signal is measured over all time, i.e., $T \rightarrow \infty$

so

$$\text{average power} = \lim_{T \rightarrow \infty} \left(\frac{1}{T} \int_{t=-T/2}^{T/2} y^2(t) dt \right)$$

Energy Signal

- To get energy in Joules (J), it is necessary to integrate over some specified time interval.
- So the energy in a signal between time 0 and time T :

$$\text{energy}(T) = \int_{t=0}^T y^2(t) dt$$

- and thus the total energy is

$$\text{total energy} = \int_{t=-\infty}^{\infty} y^2(t) dt$$

Relation of Power and Energy Signal

- the average power can be expressed as:

$$\text{average power} = \lim_{T \rightarrow \infty} \left(\frac{1}{T} \times \text{total energy} \right)$$

- Clearly, because of the $1/T$ factor, if the total energy is finite then the average power is zero.
- Conversely, if the average power is not zero then the total energy is infinite

Energy Signal

- if the resistor is 1Ω , the power is equal to the square of the voltage or current signal. In general, the instantaneous power of a signal is taken to be the square of the signal:

$$p(t) = y^2(t)$$

- The average power of a signal is its mean value. For instance, over a time period $-T/2$ to $T/2$

$$\text{average power} = \frac{1}{T} \int_{t=-T/2}^{T/2} y^2(t) dt$$

Example – Finite Energy Signal

- Consider a transient signal that starts at $t = 0$ and decays to zero with an exponential form:

$$y(t) = e^{-t}, \quad t \geq 0$$

$$\text{total energy} = \int_{t=-\infty}^{\infty} y^2(t) dt = \int_{t=0}^{\infty} e^{-2t} dt = \frac{1}{2}$$

which is a finite energy signal

Example - Non-zero Average Power Signal

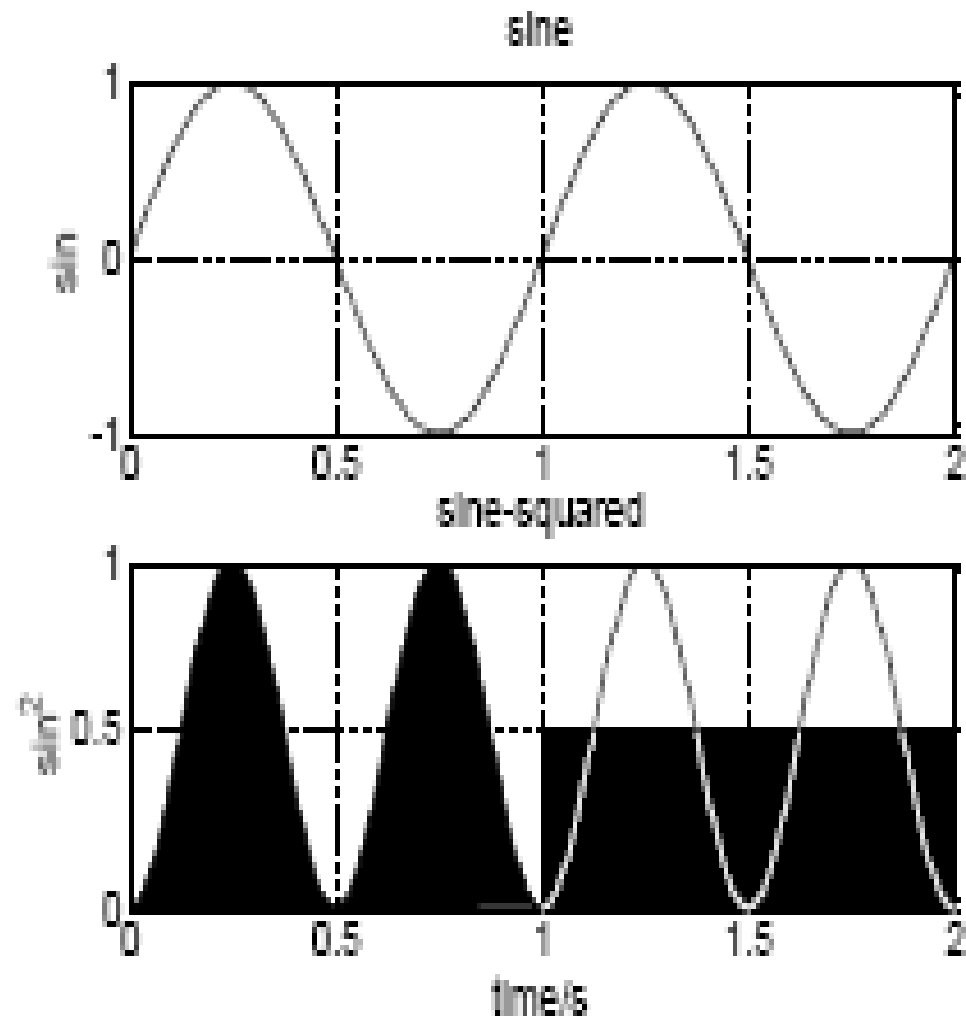
- The average power of a periodic signals can be written as, (WHY?)

$$\bar{P} = \frac{1}{T} \int_{t=0}^T y^2(t) dt$$

- Let $y(t) = \sin(\omega t)$, where T is the period equals $2\pi/\omega$

$$\bar{P} = \frac{1}{T} \int_{t=0}^T \sin^2\left(\frac{2\pi}{T}t\right) dt = \frac{1}{2}$$

Example



Deterministic Signals

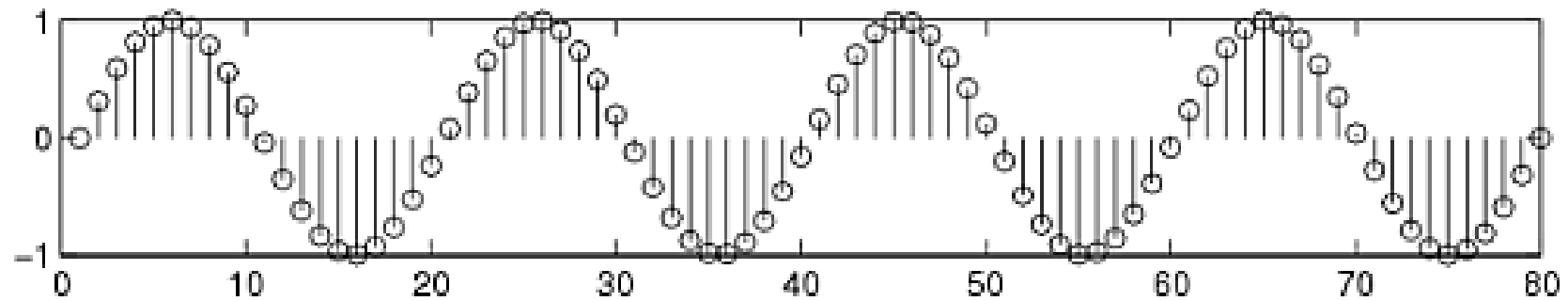
- Everything is known about the signal
- $x(t) = A\sin(\omega t)$, $\{A, \omega\}$ known parameters, no noise or unknown parameters

Random Signals

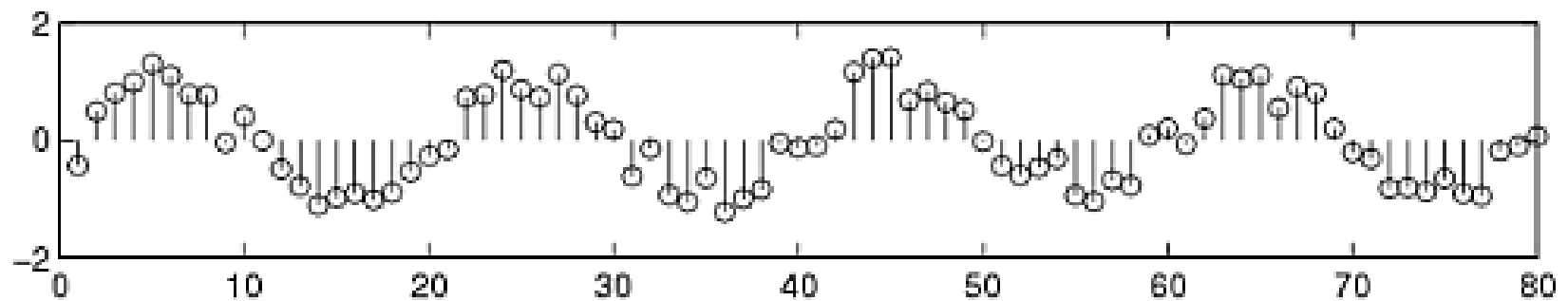
- Some property or parameter of the signal is unknown
- May be completely random such as additive noise $n(t)$
- random process
- Deterministic signal with additive noise eg. $r(t) = x(t) + n(t)$
- Random parameter of a deterministic signal eg. $x(t) = A\sin(\omega t)$, A is random number
- $r(t)$, $n(t)$, $x(t)$ all random processes

Signal Classification- Deterministic vs Random

Deterministic

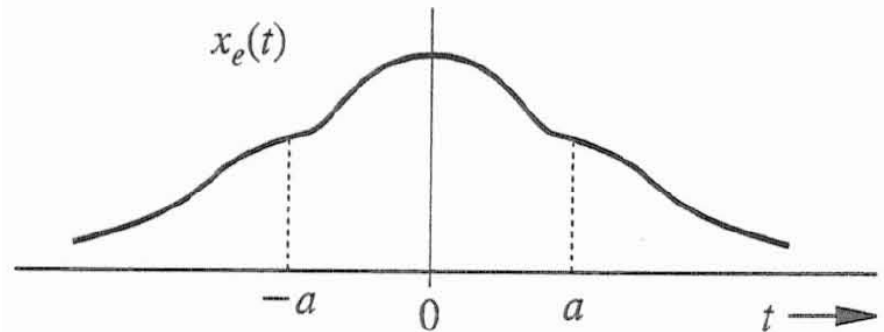


Random



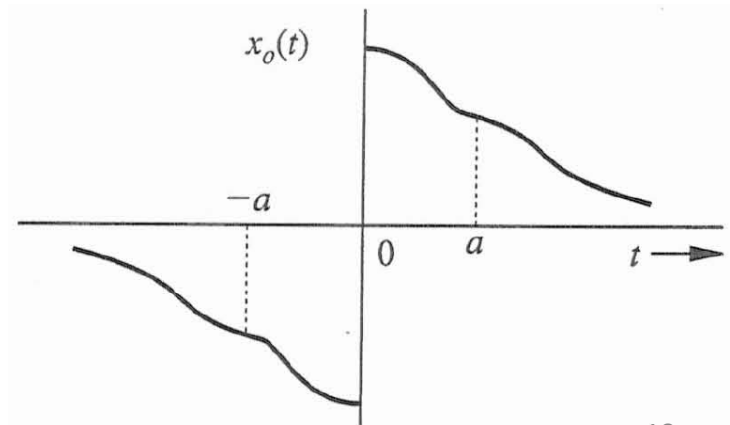
A real function $x_e(t)$ is said to be an even function of t if

$$x_e(t) = x_e(-t)$$

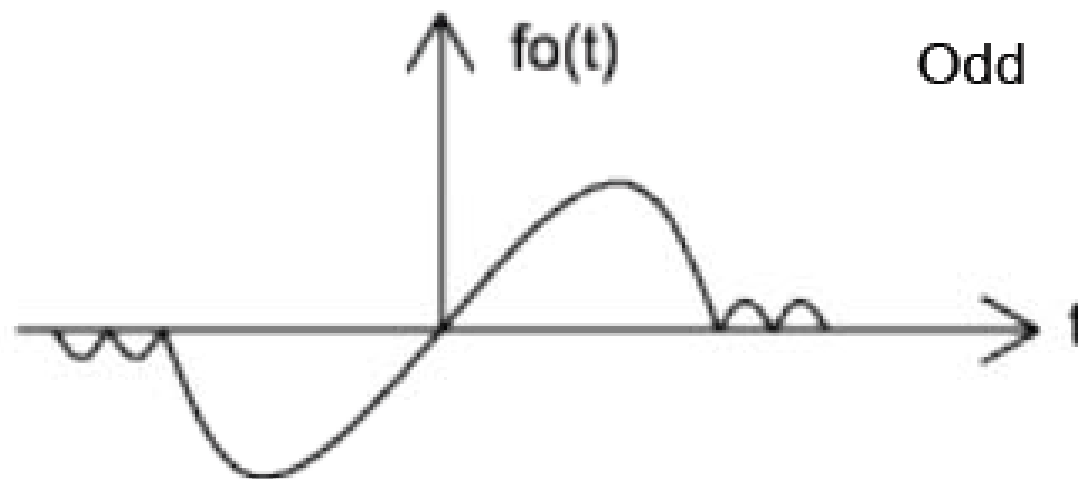
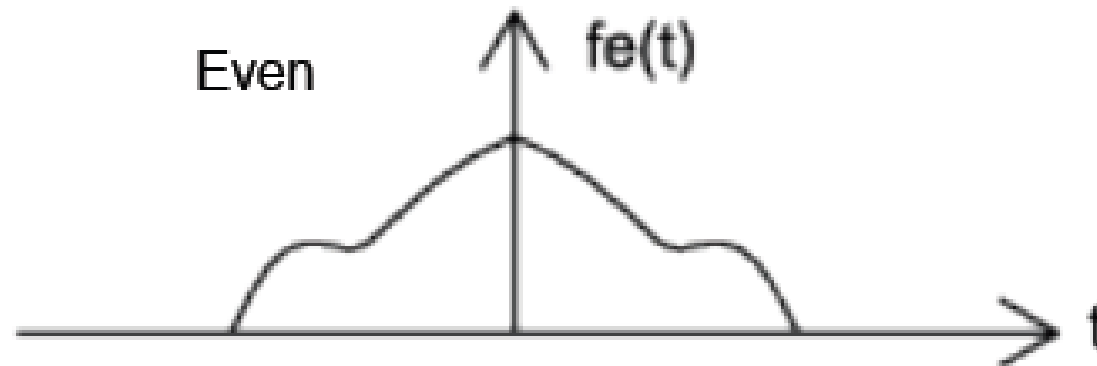


A real function $x_o(t)$ is said to be an odd function of t if

$$x_o(t) = -x_o(-t)$$



Signal Classification- Even vs Odd



Every signal $x(t)$ can be expressed as a sum of even and odd components because:

$$x(t) = \underbrace{\frac{1}{2}[x(t) + x(-t)]}_{\text{even}} + \underbrace{\frac{1}{2}[x(t) - x(-t)]}_{\text{odd}}$$